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A CONTRIBUTION TO THE THEORY OF A BENT FLAME

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Within the framework of the Zel'dovich–Frank-Kamenetskii theory of a laminar flame, an eigenvalue linearized stationary boundary-value problem is considered to establish the dependence of the flame velocity on the curvature of its front of sinusoidal shape. The analysis has been carried out for arbitrary Lewis numbers. Various mechanisms contributing to the smoothing out of the bent front of the flame are considered.

One of the most puzzling natural phenomena is spontaneous turbulization of a flame in gas mixtures. Two types of instability are known that might lead to the incipience of this regime of combustion: hydrodynamic [1, 2] and diffusional-thermal [3, 4]. In the case of generally formulated problem of stability, the behavior of combustion is considered in relation to arbitrary disturbances that cause the bending of a plane laminar flame moving with velocity v_n relative to the initial combustible mixture. In turn, the bending of the combustion front leads to a new value of the normal velocity of flame propagation, which depends on the curvature of its front. This means that now the velocity of the flame v_n is no longer a constant, as is generally assumed, of the given combustible mixture but is also related to the newly arising conditions of combustion. In the present context, it is related to the combustion-front curvature. The idea of the possible dependence of the flame velocity on the curvature of its front was proposed by Markstein [2, 4] to explain the stability of a flame with respect to hydrodynamic disturbances.

The first attempt at a theoretical analysis of the given problem was made in [3] and in an extended form was described in [4]. It was assumed in these works that the dependence of the velocity of a flame on the curvature of its front must be evident, as an indirect result, from the analysis of diffusional thermal instability. However, although the needed formula was given, no mention was made of the technique of its derivation.

The Landau–Darrier theory of hydrodynamic instability with a paradoxical conclusion on the impossibility of the existence of a laminar flame calls into question the validity of the Zel'dovich–Frank-Kamenetskii theory [5] constructed on the basis of the fundamental ideas advanced by A. N. Kolmogorov, I. G. Petrovskii, and N. S. Piskunov [4]. If the Zel'dovich–Frank-Kamenetskii theory fails to explain the stability of a laminar flame with respect to hydrodynamic disturbances, then it must be admitted that this elegant and physically deep theory is incomplete. In essence, the problem amounts to the determination of the form of the so-called Markstein constant, which is the coefficient of the curvature in the expression for the velocity of a bent flame. The value he suggested for this coefficient ensures hydrodynamic stability of a laminar flame up to a Reynolds number of Re $\sim 10^2$. On the other hand, in experiments, the stability is observed up to Re $\sim 10^4$.

Formulation of the Mathematical Problem. Mathematically speaking, we are to solve the eigenvalue boundary-value problem. Physically, we are to find a new mode of burning that changes the former

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stationary regime on introduction of a stationary (and generally nonstationary) disturbance that led to the bending of the laminar combustion front.

We will note the fundamental differences between Markstein's method of finding the length (the method given in the present work) and that suggested earlier in [3]. These differences are:

(1) the velocity of flame propagation is the eigenvalue of the boundary-value problem not only for a plane combustion front but also in the general case of a bent front;

(2) investigation of diffusional-thermal instability and derivation of the flame velocity as a function of the curvature of its front are problems different in physical content.

The plane stationary front of the flame in a gas mixture with a chemical reaction of the first order proceeding in it is described by the system of equations [4]

$$v_{\rm n}\frac{dT}{dx'} = \kappa\frac{d^2T}{dx'^2} + \frac{Q}{c_p}Nk_0\exp\left(-\frac{E}{RT}\right),\quad v_{\rm n}\frac{dN}{dx'} = D\frac{d^2N}{dx'^2} - Nk_0\exp\left(-\frac{E}{RT}\right)... \tag{1}$$

The introduction of the dimensionless parameters

$$u = \frac{T-T_0}{T_{\rm b}-T_0} = \frac{c_p}{QN_0}(T-T_0)\,,\quad b = \frac{N_0-N}{N_0}\,,\quad x = \frac{x'v_*}{\kappa}\,,\quad w = \frac{v_{\rm n}}{v_*}\,,\quad {\rm Le} = \frac{D}{\kappa}\,,\quad T_{\rm b} = T_0 + \frac{Q}{c_p}N_0\,,$$

where v_* is the velocity scale defined below, converts system (1) into

$$w\frac{du}{dx} = \frac{d^2u}{dx^2} + W\,,\quad w\frac{db}{dx} = {\rm Le}\frac{d^2b}{dx^2} + W\,,\quad W = \frac{k_0\kappa}{v_*^2}(1-b)\exp\left(-\frac{E}{R\,[T_0+u\,(T_{\rm b}-T_0)]}\right). \tag{2}$$

The following boundary conditions correspond to the problem of flame propagation according to Eq. (2):

$$x\to-\infty:\quad u=b=0\,;\quad x\to+\infty:\quad du/dx = db/dx = 0\,. \tag{3}$$

Now, on the plane front we superimpose a small disturbance that causes the same small deformation of the front over the transverse coordinates *y*, *z*. From the physical point of view, this changes little the spatial distribution of the velocity of the combustible mixture gas that impinges on the stationary plane front of the flame, and a new stationary state of the combustion process develops. Then, accurate to the small second-order values in the degrees of disturbances, the system of equations that describe the combustion front takes the following form:

$$w\frac{\partial u}{\partial x} = \Delta u + W\,,\quad w\frac{\partial b}{\partial x} = {\rm Le}\,\Delta b + W\,, \tag{4}$$

where Δ is the Laplace operator in the Cartesian coordinate system and W has not been linearized as yet to make the equations shorter. We denote the solutions of system (2) and (3) by $u^0(x)$, $b^0(x)$, and w^0. The solution of Eqs. (4) for a slightly bent front of the flame will be sought in the form

$$u = u^0(x) + V(x,y,z)\,,\quad b = b^0(x) + P(x,y,z)\,,\quad w = w^0 + w'\,, \tag{5}$$

where V and P are new independent functions. The substitution of Eq. (5) into Eq. (4), subsequent linearization, and the use of Eq. (2) in intermediate manipulations after simple transformations leads to the equations for V and P:

$$\frac{du^0}{dx} w' = \Delta V - w^0 \frac{\partial V}{\partial x} + \frac{\partial W}{\partial u^0} V + \frac{\partial W}{\partial b^0} P ,$$

$$\frac{db^0}{dx} w' = \text{Le}\, \Delta P - w^0 \frac{\partial P}{\partial x} + \frac{\partial W}{\partial u^0} V + \frac{\partial W}{\partial b^0} P ,$$

$$\Delta' = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} . \quad (6)$$

In Eqs. (6), the function W depends only on $u^0(x)$ and $b^0(x)$.

After that, system (6) is transformed as follows. The quantity w' is presented in the form $w' = -q\Delta'\xi$, where q = const is the Markstein constant in dimensionless form and ξ is the deformation of the combustion front. For the functions V, P, and ξ we adopt

$$V = \xi F(x) , \quad P = \xi G(x) , \quad \Delta'\xi = -\lambda^2\xi , \quad \lambda^2 = \lambda_1^2 + \lambda_2^2 ,$$

where λ_1 and λ_2 are the wave numbers in the y and z directions. As a result, Eq. (6) is transformed into

$$\frac{d^2F}{dx^2} - w^0 \frac{dF}{dx} - \lambda^2 F + \frac{\partial W}{\partial u^0} F = -\frac{\partial W}{\partial b^0} G + q\lambda^2 \frac{du^0}{dx} ,$$

$$\text{Le}\,\frac{d^2G}{dx^2} - w^0 \frac{dG}{dx} - \lambda^2\, \text{Le}\, G + \frac{\partial W}{\partial b^0} G = -\frac{\partial W}{\partial u^0} F + q\lambda^2 \frac{db^0}{dx} . \quad (7)$$

We assume that the parameter λ is prescribed. Then this necessitates the specific representation of the eigenvalue of w: $w = w^0 - q\Delta'\xi$ from Eqs. (4). In the given form of w, the constant q is the eigenvalue of Eqs. (7) with corresponding boundary conditions whose formulation will be given below. But first we consider a plane flame.

Plane Flame. The task of solving Eqs. (7) with the Arrhenius law (the third relation in (2)) is mathematically very formidable. It can be simplified substantially by using W in the form

$$W = \frac{k_0\kappa}{v_*^2} (1 - b) \exp\left(-\frac{E}{RT_{\text{b}}}\right) \eta\, (u - u_*) , \quad (8)$$

where η is the Heaviside unit function and u_* is the dimensionless temperature whose explicit form can be determined by requiring the coincidence between the form of the flame velocity asymptotically in the limit $u_* \to 1$ with the known result of the Zel'dovich–Frank-Kamenetskii theory.

The given form of W preserves the main properties of the Arrhenius law: its nonlinearity and strong dependence on temperature. It is clear that in contrast to analytical investigations [3, 4], where W was represented in the form of the Dirac δ-function, the form (8) has more information about the character of the occurring chemical reaction.

Positioning the place of discontinuity (8) at the point $x = 0$ and equating the subscripts 1 and 2 to the temperature u and the burning-out b respectively for $x < 0$ and $x > 0$, we obtain the distributions $u^0(x)$ and $b^0(x)$:

$$x < 0 : \quad u_1^0 = u_* \exp(w^0 x) = \frac{k}{k + w^0} \exp(w^0 x) ;$$

$$b_1^0 = \left(1 - \frac{w^0 k}{a}\right) \exp\,(w^0 x/\mathrm{Le}) = \frac{k^2\,\mathrm{Le}}{a} \exp\,(w^0 x/\mathrm{Le})\,,$$

$$x > 0: \quad u_2^0 = 1 - \frac{w^0}{k + w^0} \exp\,(-kx)\,, \quad b_2^0 = 1 - \frac{w^0 k}{a} \exp\,(-kx)\,, \quad k = \frac{\sqrt{(w^0)^2 + 4a\,\mathrm{Le}} - w^0}{2\,\mathrm{Le}}\,, \tag{9}$$

$$a = \frac{k_0 \kappa}{v_*^2} \exp\left(-\frac{E}{RT_\mathrm{b}}\right), \quad (w^0)^2 = \left(\frac{1 - u_*}{u_*}\right)^2 \frac{a}{\mathrm{Le} + (1 - u_*)/u_*}\,,$$

where k is the positive root of the equation $\mathrm{Le}\,k^2 + w^0 k - a = 0$.

Solutions (9) satisfy boundary conditions (3), the continuity conditions $u^0(x)$ and $b^0(x)$, and their first derivatives at the point $x = 0$.

In the limit, $a \to \infty$ (then $u_* \to 1$). If we adopt that

$$\frac{1 - u_*}{u_*} \approx 1 - u_* = \frac{T_\mathrm{b}}{T_\mathrm{b} - T_0} \sqrt{\frac{2T_0}{T_\mathrm{b}} \frac{RT_\mathrm{b}}{E}}\,,$$

we obtain an expression for the flame velocity v_n given in [4]. It is convenient to take v_n as the velocity scale v_*. Then

$$w^0 = 1\,, \quad a = \mathrm{Le}\,n \left(\frac{n-1}{n} \frac{E}{RT_\mathrm{b}}\right)^2, \quad n = \frac{T_\mathrm{b}}{T_0}\,. \tag{10}$$

With account for the above, we will assume in what follows that $w^0 = 1$.

With the accepted form of the ignition temperature u_* in the limit $E/RT_\mathrm{b} \to \infty$ the rate of chemical reaction in the form of (8) tends to the Dirac δ-function and arbitrarily exactly approximates a similar expression according to the Arrhenius law, since the solutions of Eqs. (2) and (3) in this limit coincide for both types of the reaction rate.

Dependence of the Flame Velocity on the Front Curvature. Using Eqs. (9) in Eqs. (8), we have

$$\frac{d^2F}{dx^2} - \frac{dF}{dx} - \lambda^2 F + a\,(1 + b^0)\,\delta\,(u^0 - u_*)\,F = a\eta\,(u^0 - u_*)\,G + q\lambda^2 \frac{du^0}{dx}\,,$$

$$\mathrm{Le}\,\frac{d^2G}{dx^2} - \frac{dG}{dx} - \lambda^2\,\mathrm{Le}\,G - a\eta\,(u^0 - u_*)\,G = -\,a\,(1 - b^0)\,\delta\,(u^0 - u_*)\,F + q\lambda^2 \frac{db^0}{dx}\,. \tag{11}$$

We formulate the boundary conditions for system (11). It follows from (3) that F and G vanish when $x \to -\infty$, and their first derivatives do the same when $x \to +\infty$. The values of u, b, du/dx, and db/dx must be continuous at the point $x \approx 0$. The condition of the continuity of u and b yields the equalities

$$F_1 = F_2\,, \quad G_1 = G_2\,. \tag{12}$$

The condition of the continuity of their derivatives (more precisely, of energy and mass fluxes) leads to the requirements

$$\frac{d^2u_1^0}{dx^2} + \frac{dF_1}{dx} = \frac{d^2u_2^0}{dx^2} + \frac{dF_2}{dx}\,, \quad \frac{d^2b_1^0}{dx^2} + \frac{dG_1}{dx} = \frac{d^2b_2^0}{dx^2} + \frac{dG_2}{dx}\,. \tag{13}$$

Since $u^0(x)$ and $b^0(x)$ are the solutions of the equations

$$\frac{du^0}{dx} = \frac{d^2u^0}{dx^2} + a\,(1 - b^0)\,\eta\,(u^0 - u_*)\,, \quad \frac{db^0}{dx} = \mathrm{Le}\,\frac{d^2b^0}{dx^2} + a\,(1 - b^0)\,\eta\,(u^0 - u_*)\,,$$ @ABSATZ0 = indicating the presence of the discontinuity in the second derivatives of $u^0(x)$ and $b^0(x)$ at the point $x = 0$

$$\frac{d^2u_1^0}{dx^2} - \frac{d^2u_2^0}{dx^2} = k\,, \quad \frac{d^2b_1^0}{dx^2} - \frac{d^2b_2^0}{dx^2} = \frac{k}{\mathrm{Le}}\,,$$

conditions (13) take the following final form:

$$\frac{dF_1}{dx} - \frac{dF_2}{dx} + k = 0\,, \quad \frac{dG_1}{dx} - \frac{dG_2}{dx} + \frac{k}{\mathrm{Le}} = 0\,. \tag{14}$$

In addition to this, the first derivatives of $F(x)$ and $G(x)$ themselves are discontinuous at the point $x = 0$, as follows directly from Eq. (11). Integrating Eq. (11) over the vanishingly small region near $x = 0$ and using the known [6] properties of the δ-function, we find

$$\frac{dF_2}{dx} - \frac{dF_2}{dx} + (k + 1)\,F_2 = 0\,, \quad \frac{dG_2}{dx} - \frac{dG_1}{dx} + \frac{k + 1}{\mathrm{Le}}\,F_2 = 0\,. \tag{15}$$

But one of the relations in (15) is extraneous, since the second expressions in (14) and (15) lead to the equality

$$F_2 = -\,\frac{k}{k + 1} = F_1\,. \tag{16}$$

This, together with the first relation in (15), leads to the already available first condition from Eq. (14). One of the equalities of (16) can be taken as a boundary condition which will be supplementary to Eqs. (14) and (15). Thus, when $x = 0$, we have five boundary conditions for finding the four unknown integration constants (after the conditions for $x \to \pm\infty$ are satisfied) and the eigenvalue q.

We now begin solution of Eqs. (11). For the region with $x < 0$ from Eq. (11), with Eq. (10) taken into account, we obtain the system of equations

$$\frac{d^2F_1}{dx^2} - \frac{dF_1}{dx} - \lambda^2 F_1 = q\lambda^2\,\frac{k}{k + 1}\,\exp(x)\,, \quad \mathrm{Le}\,\frac{d^2G_1}{dx^2} - \frac{dG_1}{dx} - \lambda^2\,\mathrm{Le}\,G_1 = q\lambda^2\,\frac{k^2}{a}\,\exp(-kx)\,.$$

Its solutions that vanish for $x \to -\infty$ are the following:

$$F_1 = f_1\,\exp(\alpha x) - \frac{qk}{k + 1}\,\exp(x)\,, \quad \alpha = \frac{1 + \sqrt{1 + 4\lambda^2}}{2}\,;$$

$$G_1 = g_1\,\exp(\beta x) - \frac{qk^2}{a\,\mathrm{Le}}\,\exp(x/\mathrm{Le})\,, \quad \beta = \frac{1 + \sqrt{1 + 4\lambda^2\,\mathrm{Le}^2}}{2\,\mathrm{Le}}\,. \tag{17}$$

Similarly, in the region with $x > 0$ we have

$$\frac{d^2F_2}{dx^2} - \frac{dF_2}{dx} - \lambda^2 F_2 = aG_2 + q\lambda^2 \frac{k}{k+1} \exp(-kx) , \quad \mathrm{Le}\frac{d^2G_2}{dx^2} - \frac{dG_2}{dx} - \lambda^2 \,\mathrm{Le}\, G_2 - aG_2 = q\lambda^2 \frac{k^2}{a} \exp(-kx) ,$$

$$G_2 = g_2 \exp(-\chi x) - \frac{qk^2}{a\,\mathrm{Le}} \exp(-kx) , \quad \chi = \frac{-1 + \sqrt{1 + 4\,\mathrm{Le}\,(\lambda^2\,\mathrm{Le} + a)}}{2\,\mathrm{Le}} ,$$

$$F_2 = f_2 \exp(-\gamma x) + A_1 g_2 \exp(-kx) - qA_2 \exp(-\chi x) ; \quad \gamma = \frac{-1 + \sqrt{1 + 4\lambda^2}}{2} , \tag{18}$$

$$A_1 = \frac{a}{\chi^2 + \chi - \lambda^2} , \quad A_2 = \frac{k}{k+1} \frac{k^2 + k - \mathrm{Le}\,\lambda^2}{k^2 + k - \lambda^2} .$$

In Eqs. (17) and (18), the quantities f_1, f_2, g_1, and g_2 are integration constants.

The substitution of Eqs. (17) and (18) into boundary condition (12) leads to the algebraic equations

$$f_1 - q\frac{k}{k+1} = f_2 + g_2 A_1 - \frac{q}{\mathrm{Le}} A_2 , \quad g_1 = g_2 . \tag{19}$$

The use of Eqs. (17) and (18) in Eq. (14) yields

$$\alpha f_1 - q\frac{k}{k+1} + \gamma f_2 + \chi g_1 A_1 - \frac{q}{\mathrm{Le}} kA_2 + k = 0 ,$$

$$\beta g_1 - q\frac{k^2}{a\,\mathrm{Le}^2} + \chi g_2 - q\frac{k^3}{a\,\mathrm{Le}} + \frac{k}{\mathrm{Le}} = (\beta + \chi)\, g_1 - \left(\frac{q}{\mathrm{Le}} - 1\right)\frac{k}{\mathrm{Le}} = 0 , \tag{20}$$

where for the second expression the second equality of Eqs. (19) was used and also (see Eq. (9)) $\mathrm{Le}\; k^2 + k - a = 0$.

Taking into account the second equality of Eqs. (19), we find

$$g_1 = \frac{k}{\mathrm{Le}\,(\chi + \beta)}\left(\frac{q}{\mathrm{Le}} - 1\right). \tag{21}$$

From condition (16) we define f_1:

$$f_1 = \frac{k}{k+1}(q-1) . \tag{22}$$

Simple calculations from the first relations of Eqs. (19), (20), and (21), (22) yield the equation

$$\frac{k}{k+1}[q(\alpha - 1) - \alpha - \gamma] + \frac{\chi - \gamma}{\chi + \beta}\frac{k(\mathrm{Le}\,k + 1)}{\chi^2 + \chi - \lambda^2}\frac{k}{\mathrm{Le}}\left(\frac{q}{\mathrm{Le}} - 1\right) - \frac{k(k-\gamma)}{k+1}\frac{k^2 + k - \mathrm{Le}\,\lambda^2}{k^2 + k - \lambda^2}\left(\frac{q}{\mathrm{Le}} - 1\right) + k = 0 . \tag{23}$$

In this expression, we let the parameter

$$k \approx \sqrt{\frac{a}{\mathrm{Le}}} = \sqrt{n}\,\frac{n-1}{n}\,\frac{E}{RT_{\mathrm{b}}}$$

go to infinity, i.e., we consider the case of large activation energy. With this limit the terms proportional to k cancel out. Neglecting the small values of the order of $1/k$ and above, we have

$$q = \mathrm{Le}\,\frac{1/\mathrm{Le} - \beta + \lambda^2/\alpha}{1/\mathrm{Le} - \beta + \mathrm{Le}\,\lambda^2/\alpha} + o\,(k^{-1})\,.$$

From this it is seen that when Le = 1, q = 1 for any wave numbers. But if $\mathrm{Le} \neq 1$, then in the limit $\lambda \to 0$; when the bent front of the flame is degenerated into a plane surface, then $q \sim \lambda^{-2} \to \infty$, which has no physical meaning. Therefore, stationary solution of problem (2), (3) with a slightly bent front is possible only in the case of Le = 1. This result remains valid also for an arbitrary value of k, as can be easily verified by assuming that Le = 1 in Eq. (23). Taking into account that $\alpha = \gamma + 1$ also when Le = 1 $\chi(\chi + 1) = k^2 + k + \lambda^2$, $\beta = \alpha$, we obtain

$$(q - 1)\left(\frac{2\gamma - k}{k + 1} + \frac{\chi - \gamma}{\chi + \gamma + 1}\right) = 0\,.$$

The expression within the second parentheses does not vanish for arbitrary values of k and λ; therefore, we obtain the well-known result that q = 1 [4].

The natural question suggesting itself here is the diffusional-thermal stability of the solutions found. The necessary, but, most likely, insufficient condition of stability was formulated, even in [3], on the basis of clear physical arguments: $q > 0$. It is possible that the value q = 1 found in the present work does not guarantee the complete stability of solutions (17) and (18). It is still necessary to find a more exact condition of stability.

Discussion of the Results Obtained. Thus, in contradiction to the experiment, the investigation carried out demonstrated the impossibility of the existence of a stationary arbitrarily bent flame at arbitrary Lewis numbers: there is not only a slightly bent flame in experiments, but also a strongly bent one (which, of course, is described already by the nonlinear theory). Therefore, without invoking additional principles, the Zel'dovich–Frank-Kamenetskii theory of laminar combustion of gases does not yield the dependence of the flame rate on the front curvature, which would make it possible to resolve the paradox of the hydrodynamic instability of flame. Moreover, there is no linear dependence, similar to that from the Markstein theory, of the flame rate on the front curvature, except for the case of Le = 1.

Returning to the problem of hydrodynamic instability of a laminar flame, we see that with the existing notions on combustion of gas mixtures this problem cannot be solved completely.

The contradictions between the theory and experiment noted in the previous section can be removed if we assume that in the stationary flame observed in the experiments a chemical reaction proceeds with an effective Lewis number equal to unity. This assumption can be substantiated by the fact that the description of combustion, used in the present work, by the Zel'dovich–Frank-Kamenetskii model that admits arbitrary Lewis numbers is very simplified. In all probability there are physical principles forbidding the use of Lewis numbers other than equal to unity in simulation of combustion of gas mixtures by means of a simple molecular reaction.

In addition to [2], an attempt at a theoretical determination of q was also undertaken in [7], where q = 1 was found in Markstein's formulas $w' = -q\Delta'\xi$. The evolution of cellular structures assigned with the initial profile is investigated numerically in [8] with allowance for the gas viscosity at an infinitely large activation energy. But from the very beginning the normal velocity of the flame in the formulation of the problem in [8] (just as in [2, 4, 7]) is assumed to be constant and independent of the front curvature. Nevertheless, in [8] a stable cellular structure does manifest itself, and this is attributed in [8] to the influence of nonlinear effects, i.e., the results of [8] point to the possibility of explaining the cellular structure of the flame without resorting to Markstein's formula.

In [9, 10], a theoretical study of the cellular structure of flame on the basis of Sivashinskii's equation is carried out, assuming a constant rate of flame on bending of its front. Because of the presence of different mechanisms that lead to the appearance of terms of the form $q\Delta'\xi$, they are usually taken as the dependence of the rate of a flame on the curvature. Taking an elementary example [11], we will show in which way the terms proportional to the second derivative with respect to the spatial variable can appear because of the displacement ξ of the flame front. Let Eq. (4) in a nonstationary form be $E/RT_b = \infty$, Le = 1 ($u = b$), and the dependence of the temperature and burning-out over the coordinate z be absent:

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\,. \tag{24}$$

Then, instead of Eq. (5) we have

$$u = u^0(x) + \xi(y)\frac{du^0}{dx}\,, \tag{25}$$

where $u^0(x)$ is the Michelson distribution:

$$u^0(x) = \exp(w^0 x)\,, \quad x < 0\,; \quad u^0(x) = 1\,, \quad x > 0\,.$$

Assuming the flame rate to be independent of the curvature of its front and substituting (25) into (24), we obtain the equation

$$\frac{\partial \xi}{\partial t} = \frac{\partial^2 \xi}{\partial y^2}\,, \tag{26}$$

which is the linear part of Sivashinskii's equation [9, 10] and which describes the smoothing of inhomogeneities of the flame front owing to the dissipative processes of diffusion and heat conduction. Moreover, if the gas-velocity is taken to be variable (in the former coordinate system associated with a nonperturbed flame), for example, due to the hydrodynamic drop of pressure, then on the left-hand side of Eq. (26) gas-velocity perturbation appears additionally and Eq. (26) transforms into the boundary condition from Markstein's theory [2] of hydrodynamic instability of a laminar flame. The distribution of temperature (25) means constant density on both sides of the flame front, in agreement with the physical content of hydrodynamic instability; pressure perturbations are associated in this case with the perturbations of the gas velocity, rather than temperature and density [1].

CONCLUSIONS

1. Markstein's linear dependence (with the possible existence of a nonlinear dependence not being negated) of the flame rate, with the one-stage molecular reaction proceeding in it, on the flame front curvature is possible only at a Lewis number equal to unity.

2. There are two mechanisms favoring the smoothing of the bent flame front: the first is directly associated with the dissipative processes of reacting gas diffusion and heat conduction and the other with the dependence of the flame rate on the curvature of its front; the second mechanism is indirectly associated with the dissipative processes.

NOTATION

T and u, dimensional and dimensionless temperatures; N, concentration of reacting substance; Q, thermal effect of a chemical reaction; E, activation energy; R, universal gas constant; k_0, pre-exponential factor; v_n and w, dimensional and dimensionless velocities of flame motion relative to the initial combustible mixture; c_p, heat capacity of a gas mixture at constant pressure; D and κ, diffusion coefficient and thermal diffusivity; x', coordinate; T_0 and N_0, initial values of the temperature and concentration of a reacting substance; w', change in the flame velocity as a result of the bending of its front; T_b, (temperature of burning) adiabatic temperature of a flame; W, chemical reaction rate; Le, Lewis number. Subscripts: n, normal; b, burning.

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